Instructions: Complete each of the following exercises for practice.

1. Use Stokes's Theorem to evaluate $\iint_S \text{curl}(\mathbf{F}) \cdot d\mathbf{S}$ for S oriented upward.
   (a) $\mathbf{F}(x,y,z) = \langle x^2\sin(z), y^2, xy\rangle$, S the part of the paraboloid $z = 1 - x^2 - y^2$ above the xy-plane
   (b) $\mathbf{F}(x,y,z) = \langle ze^y, x\cos(y), xz\sin(y)\rangle$, S the hemisphere of $x^2 + y^2 + z^2 = 16$ with $y \geq 0$
   (c) $\mathbf{F}(x,y,z) = \langle \arctan(x^2yz^2), x^2y, x^2z^2\rangle$, S the cone $x = \sqrt{y^2+z^2}$ with $x \leq 2$
   (d) $\mathbf{F}(x,y,z) = \langle xyz, xy, x^2yz\rangle$, S the boundary of the cube $[-1,1]^3$ minus the bottom face
   (e) $\mathbf{F}(x,y,z) = \langle e^{xy}, e^{xz}, x^2z\rangle$, S the portion of the ellipsoid $4x^2 + y^2 + 4z^2 = 4$ to the right of the xz-plane

2. Use Stokes's Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ for C oriented counter clockwise from above.
   (a) $\mathbf{F}(x,y,z) = \langle x + y^2, y + z^2, z + x^2\rangle$, C the triangle with vertices $(1,0,0)$, $(0,1,0)$, and $(0,0,1)$
   (b) $\mathbf{F}(x,y,z) = \langle 1, x + yz, xy - \sqrt{z}\rangle$, C the boundary of the plane $3x + 2y + z = 1$ restricted to the first octant
   (c) $\mathbf{F}(x,y,z) = \langle xy, yz, zx\rangle$, C the boundary of the paraboloid $z = 4 - x^2 - y^2$ intersect the first octant
   (d) $\mathbf{F}(x,y,z) = \langle 2y, xz, x + y\rangle$, C the intersection of the plane $z = y + 2$ and the cylinder $x^2 + y^2 = 1$

3. Verify Stokes's Theorem holds by computing $\int_{\partial S} \mathbf{F} \cdot d\mathbf{r}$ and $\iint_S \text{curl}(\mathbf{F}) \cdot d\mathbf{S}$ separately and comparing.
   (a) $\mathbf{F}(x,y,z) = \langle -y, x, -2\rangle$, S the cone $z^2 = x^2 + y^2$ with $0 \leq z \leq 5$ oriented downward
   (b) $\mathbf{F}(x,y,z) = \langle -2yz, y, 3x\rangle$, S the part of the paraboloid $z = 5 - x^2 - y^2$ above plane $z = 1$ oriented upward
   (c) $\mathbf{F}(x,y,z) = \langle y, z, x\rangle$, S the hemisphere $x^2 + y^2 + z^2 = 1$ with $x \leq 0$ oriented in the negative x-direction

4. Let C be an arbitrary smooth, simple, closed curve in the plane $x + y + z = 1$. Show $\int_C z\,dx - 2x\,dy + 3y\,dz$ depends only in the area enclosed by C.

5. Suppose S is a surface satisfying the hypotheses of Stokes's Theorem and f, g are functions with continuous second-order partial derivatives. Prove each of the following.
   (a) $\int_{\partial S} f\nabla g \cdot d\mathbf{r} = \iint_S (\nabla f \times \nabla g) \cdot d\mathbf{S}$
   (b) $\int_{\partial S} f\nabla f \cdot d\mathbf{r} = 0$
   (c) $\int_{\partial S} (f\,\nabla\, g + g\,\nabla\, f) \cdot d\mathbf{r} = 0$